258. Proposed by REV. R. D. CARMICHAEL, Hartselle, Ala.

Sum the infinite series $\dfrac{n^2}{(4n^2-1)^2}$ beginning with $n=1$, n being always odd.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

$$\frac{n^2}{(4n^2-1)^2}=\tfrac{1}{16}\left[\frac{1}{(2n-1)^2}+\frac{1}{(2n+1)^2}-\frac{1}{2n+1}+\frac{1}{2n-1}\right]$$

Let $u=1, 3, 5, 7, \ldots\ldots$ Then

$$\Sigma\frac{n^2}{(4n^2-1)^2}=\tfrac{1}{16}\left[\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\frac{1}{9^2}+\ldots\right]+\tfrac{1}{16}\left[\frac{1}{1}-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\ldots\ldots\right].$$

$$\therefore\ \Sigma\frac{n^2}{(4n^2-1)^2}=\tfrac{1}{16}\cdot\frac{\pi^2}{8}+\tfrac{1}{16}\cdot\frac{\pi}{4}=\frac{\pi}{64}[\tfrac{1}{2}\pi+1]$$ (See line 1, p. 41).

Also solved by G. W. Greenwood, and Henry Heaton.

CALCULUS.

133. Proposed by NELSON L. RORAY, South Jersey Institute, Bridgeton, N. J.

Evaluate $\displaystyle\int\frac{\sqrt{(1+y)}}{1+y^2}dy$.

Solution by HENRY HEATON, Belfield, N. D.

Let $1+y=z^2$; then $\displaystyle\int\frac{\sqrt{(1+y)}}{1+y^2}dy=\int\frac{2z^2dz}{z^4-2z^2+2}=\frac{1}{2a}\int\left(\frac{z}{z^2-2az+\sqrt{2}}-\frac{z}{z^2+2az+\sqrt{2}}\right)dz$, where $a=\sqrt{\dfrac{\sqrt{2}+1}{2}}$.

$$\frac{1}{2a}\int\left(\frac{z}{z^2-2az+\sqrt{2}}-\frac{z}{z^2+2az+\sqrt{2}}\right)dz=\frac{1}{2a}\int\left(\frac{(z-a)+a}{z^2-2az+\sqrt{2}}-\frac{(z+a)-a}{z^2-2az+\sqrt{2}}\right)dz=\frac{1}{2a}\log\sqrt{\frac{z^2-2az+\sqrt{2}}{z^2+2az+\sqrt{2}}}+\frac{1}{2\sqrt{[\sqrt{2}-a^2]}}\left(\tan^{-1}\frac{z-a}{\sqrt{[\sqrt{2}-a^2]}}+\tan^{-1}\frac{z+a}{\sqrt{[\sqrt{2}-a^2]}}\right)=\sqrt{\frac{\sqrt{2}-1}{2}}\log\sqrt{\frac{1+y-\sqrt{[2\sqrt{2}+2]}\sqrt{[1+y]}+\sqrt{2}}{1+y+\sqrt{[2\sqrt{2}+2]}\sqrt{[1+y]}+\sqrt{2}}}$$

$$+\sqrt{\frac{\sqrt{2}+1}{2}}\tan^{-1}\left(\frac{\sqrt{[2\sqrt{2}-2]}\sqrt{[1+y]}}{\sqrt{2}-1-y}\right).$$